Can Design

There are many factors to consider in food packaging, including marketing, durability, cost and materials. In this problem we minimize the cost of materials for a can.

Find the height and radius that minimizes the surface area of a can whose volume is 1 liter = 1000 cm^3.

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$$V = \pi r^2 h = 1000cm^3 \quad SA = 2\pi r^2 + 2\pi r h$$

$$\pi r h = \frac{1000}{r}$$

$$\Rightarrow SA = 2\pi r^2 + \frac{2000}{r}$$

$$\frac{d}{dr} SA = 4\pi r - \frac{2000}{r^2}$$

$$\frac{d}{dr} SA = 0 \Rightarrow 4\pi r = \frac{2000}{r^2}$$

$$4\pi r^3 = 2000$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}}$$

$$\lim_{r \to 0^+} SA = \infty$$

$$\lim_{r \to \infty} SA = \infty$$

$\Rightarrow$ r is a minimum point as there is only 1 critical point.

$$\therefore r = 5\sqrt[3]{\frac{4}{\pi}}, \quad h = 10\sqrt[3]{\frac{4}{\pi}}$$

$$\approx 5.42cm \quad \approx 10.84cm$$

$$h = \frac{1000}{\pi r^2}$$

$$= \frac{1000}{\pi \left(\frac{500}{\pi}\right)^{\frac{2}{3}}}$$

$$= \frac{2 \cdot 500^{\frac{1}{3}}}{\pi^{\frac{1}{3}}}$$

$$= 2\sqrt[3]{\frac{500}{\pi}}$$